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WINNERS, LOSERS AND MARKET IMPACT*

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Abstract

Dual trading reduces total trading volume and market depth, but has no effect on price efficiency. Trading volume and gross (of commission fees) profits of the informed traders are lower with dual trading while trading volume and gross losses of the uninformed traders are unaffected. This effect of the ban on the uninformed is the same irrespective of whether they act as noise traders or as rational, risk-averse hedgers.

Commission fees charged by the broker may increase with dual trading if the probability of a trade occurring decreases, the broker's fixed costs are large and dual trading profits are small. The utility of the uninformed traders (net of commission fees) will increase with dual trading when their expected commission costs are relatively lower and the gross profits of all other traders also increase. The net profits of the informed traders will be lower with dual trading if the probability of an informed trade increases less relative to the probability of an uninformed trade and the relative commission costs of the uninformed traders are lower.

Dual trading refers to the practice of brokers trading for their own accounts in addition to bringing their customers' orders to the market. The practice has, over the years, been controversial with proponents of dual trading vouching for its salutary effects on market liquidity and price efficiency and opponents emphasizing the potential conflicts of interest between dual trading brokers and their customers. On the regulatory front, the anti-dual-trading camp currently holds sway. In March 1993, the Commodities Futures Trading Commission (CFTC) proposed limiting dual trading in the largest futures markets. The Chicago Mercantile Exchange (CME) banned dual trading in all active contracts effective May 20, 1991. But, the issue is by no means settled, as the Exchange still faces great resentment over the ban.[1]

I develop a model to study how a ban on dual trading will affect aggregate market characteristics (total trading volume, market depth and price efficiency). I also study its distributional effects by looking at the impact on brokers' commission fees as well as the trading volumes and profits (both gross and net of commission fees) of the informed and the uninformed traders. The main conclusions are that dual trading reduces total trading volume and market depth without affecting price efficiency. Ignoring commission fees, informed traders are hurt with dual trading while uninformed traders neither gain nor lose. With commission fees, which group benefits depends upon how dual trading affects the probability of an informed trade relative to an uninformed trade and whether relative commission costs increase or decrease with dual trading.

The microstructure of the basic model follows Kyle (1985). Uninformed noise traders[2] and a group of m informed traders submit market orders to a broker who places them (along with her own orders) with a marketmaker for execution. The marketmaker batches the total order flow and executes them at a single price. The price is determined by assuming that the marketmaker makes zero profits conditional on observing the total order flow.

The broker's motive for dual trading comes from her private observations of the size of her customers' orders.[3] In equilibrium, she is able to infer all of her informed customers' information from her observations and profit through mimicking or piggybacking on the informed trades. Because their orders are now executed at a higher (in absolute value) price, the broker's piggybacking hurts the informed traders, who react by restricting their order sizes.[4] Thus, informed trading volume is higher when dual trading is banned.

The broker also offsets a portion of uninformed trading volume. This decreases total trading volume and market depth when there is dual trading. However, because volume and depth decrease in the same proportion, price informativeness and the price level are unaffected by dual trading. It follows that the profits of the informed (ignoring commission fees) are lower with dual trading. Finally, total informed trading (i.e., informed plus dual trading[5]) profits are identical across markets and, therefore, so are the losses of the noise traders.

These results are unchanged when the basic model is extended to allow for rational behavior by uninformed traders. Following Spiegel and Subrahmanyam (1992), uninformed traders (who are risk-averse) trade in order to "hedge" their endowments of shares of the risky asset. Since both market depth and net uninformed trading volume (of the uninformed traders and the broker) are lower in the same proportion, the price impact of an uninformed trade and the variance of uninformed trading profits and, therefore, the expected utilities of uninformed traders are unchanged by dual trading.

Next, I analyze trading behavior when investors have to pay commission fees to the broker whenever they place an order. The commission fees are assumed to be fixed (i.e., independent of the order size). As in Fishman and Longstaff (1992), the broker incurs fixed and variable costs of brokerage. Further, the brokerage business is competitive so that the broker's total income (trading profits plus commission income) is zero. The effect of these commission fees is to make some previously feasible trades unprofitable. Then, a ban on dual trading may actually reduce commission fees if the probability of a customer trade thereby increases and dual trading profits are small. However, if dual trading reduces the probability of a customer trade the expected commission costs of all traders will also be lower with dual trading.

Ignoring commission fees, informed gross profits are lower and expected utilities of the uninformed are the same with dual trading. With commission fees, informed profits improve with dual trading if the uninformed pay relatively higher commission costs with dual trading and the probability of their trading decrease relative to that of the informed traders. Uninformed traders benefit with dual trading if the gross profits of all other traders to increase as well and, further, if their relative commission costs are lower.

Roell (1990) has a model of dual trading in which a broker observes the trade of some uninformed traders. Her model does not include the effect of commission fees. Informed traders have higher profits when dual trading is banned. Uninformed traders whose trades are observed by the broker have higher profits with dual trading. Those whose trades are not observed by the broker are hurt by dual trading.

In Fishman and Longstaff (1992), the broker has private information about whether her customer is informed or uninformed. Before commissions, all customers lose with dual trading. Including commission fees, dual trading benefits the uninformed traders and hurts the informed traders. In contrast to this paper, they assume that trading volume is fixed at one unit. As a result, the informed trader in their model fails to take into account the broker's mimicking behavior when formulating her optimal trading strategy. A further implication of this assumption is that the broker's commission fees are always lower with dual trading. They also do not model the behavior of the customer when she is uninformed. On the other hand, they allow the customer and the dual trader to trade at different prices and they also model the effect of frontrunning by the broker.

Chang and Locke (1992) find no positive correlation between the order imbalances of dual traders and customers in the currency futures market. They conclude that piggybacking behavior by dual traders is absent. However, since no distinction is made between informed and uninformed customer trades, the lack of a positive correlation is not conclusive. Another interesting result is that dual traders do not, on average, make more profits on their personal trades relative to locals (who are not privy to any information regarding customer identity or trade size). Studies also find that restricting dual trading either has little effect on bid-ask spreads (Chang and Locke (1992), CFTC (1989)) or depth (Park and Sarkar (1992)) or leads to a decrease in the realized spread (Smith and Whaley (1990), Walsh and Dinehart (1991)). Further, Park and Sarkar (1992) find that a restriction on dual trading in the S&P 500 futures market decreased total trading volume by about 4.59%. The empirical results, therefore, suggest that dual trading has no strong positive or negative market impact--perhaps because it is not an important provider of liquidity in many Chicago futures markets (as suggested by Chang and Locke (1992)).

Sections I and II develop the basic dual trading model with noise traders, ignoring commission fees and compares it to a model where dual trading is completely banned. Results on the market impact of dual trading are obtained. Section III extends the model to allow for rational behavior by uninformed traders. Section IV introduces the effect of fixed commission fees on the informed and uninformed traders' optimal trading strategies. Then, the effect of dual trading on commission fees and traders' net profits is explored in Section V. The study concludes in Section VI. All proofs are contained in the appendix.

I. THE DUAL TRADING MODEL AND SOLUTION

A. The Dual Trading Model

I consider a market in which a single risky asset with unknown liquidation value v is traded. There is a group of m informed traders each of whom receive, prior to trading, signals s^i about the unknown value v. The signals are of the form $s^i = v + e^i$, $i=1,\dots,m$ where the error terms e^i are independent of each other. In addition, there is a group of uninformed noise traders who trade for liquidity reasons. Initially, the uninformed traders' motives for trading are not modelled. Later, the basic model is extended to allow for rational behavior by the uninformed traders.

Each informed trader $i = 1,\dots,m$ submits a market order x_d^i to a broker. The noise traders also collectively submit market orders worth u to the same broker. The latter then places the total of the submitted orders (x_d+u), where $x_d = \sum_{i=1}^{m} x_d^i$, to a marketmaker for execution.[6] In the dual trading model, the broker may also trade an amount d for her own account. She may want to do so because, by observing the market orders x_d^i of the informed, she is able to infer some or all of their information s^i. In addition, it may be profitable for the broker to take the opposite position of the group of uninformed traders. The act of dual trading makes the broker both a de facto informed trader as well as an uninformed trader.[7] At this stage, I ignore commission fees (which are introduced in Section IV).

It is assumed that, when the broker places her customers' order with the marketmaker, she simultaneously sends along her own order d as well. The marketmaker then fixes a single price p_d at which she will execute the total order flow $y_d = x_d + d + u$. Following Kyle (1985), the marketmaker is assumed to be risk-neutral and competitive. Conditional on observing y_d, she earns zero expected profits.

The random variables in the model are v, u and e^i, $i=1,\dots,m$. All these variables are normally distributed with zero mean and finite variances Σ_v, Σ_u and Σ_e, respectively. Thus the m error terms are drawn from an identical distribution.[8] In addition, all investors follow linear trading rules $x_d^i = A_d s^i$, i=1,...,m (for the informed) and $d = B_1 x_d + B_2 u$ (for the broker). This implies that the marketmaker's pricing rule is also linear: $p_d(y_d) = \Gamma_d y_d$, where $1/\Gamma_d$ is the now-familiar market depth parameter.

There are three distinct stages to this trading game:

(1) Informed traders receive their information and decide how much they want to trade. In making this decision, each informed trader is aware that, first, she is in competition with the other informed traders and, second, that the broker will "piggyback" on the information conveyed by her trading decision. The informed traders care about the broker's piggybacking because they receive a less favorable price for their trades as a consequence. Noise traders simply submit u.

(2) The broker observes u and x_d^i and infers that each informed trader has some information s^{i*}, i=1,...,m. Based on her inferences and u, she decides to trade an amount d.

(3) The marketmaker fixes a price $p_d = \Gamma_d(x_d+u+d)$, where $p_d = E(v|y_d)$ and so $\Gamma_d = Cov(v,y_d)/Var(y_d)$.

This suggests the following solution method. Fix Γ_d and suppose that each informed trader i=1,...,m has decided to trade some amount x_d^i and uninformed traders have submitted demand u. From each x_d^i the broker infers information s^{i*}. She then chooses d to maximize her expected profits, where the expectation is taken with respect to the vector $(s^{1*},...,s^{m*},u)$. Each informed trader i then chooses x_d^i as a best response to $d(s^{1*},...,s^{m*},u)$, the rival informed traders' decisions x_d^j, $j \neq i$ and uninformed trades u. Finally, Γ_d is obtained from the optimal trading rules and the marketmaker's zero profit assumption.

Depending upon what the equilibrium beliefs of the broker are, there can be potentially many equilibria to the signalling game between the informed traders and the broker. Fortunately, in this model, the signalling game affords a unique solution: there is a single fully separating equilibrium. In other words, she informed traders' information is fully revealed to the broker and so in equilibrium $s^{i*} = s^i$, $i=1,\ldots,m$.

B. The Dual Trading Solution

First, I solve the signalling game between the informed traders and the broker. Given her observations of x_d and u, the broker chooses d to maximize her conditional expected profits given by $E(\pi|s^{1*},\ldots,s^{m*},u)$, where $\pi = (v-\Gamma_d y_d)d$. From the first-order condition, the optimal $d = [E(v|s^{1*},\ldots,s^{m*}) - \Gamma_d(x_d+u)]/2\Gamma_d$. The second-order condition is satisfied by $\Gamma_d > 0$. Define $t = \Sigma_v/(\Sigma_v+\Sigma_e)$ and note that $0 \le t \le 1$. t is a measure of the unconditional precision of s^i, $i=1,\ldots,m$. For example, if t=1 then s^i is a perfect signal. Then $E(v|s^{1*},\ldots,s^{m*}) = ts^*/Q$ where $Q = [1+t(m-1)]$ and $s^* = \sum_{i=1}^{m} s^{i*}$. Therefore, the broker's optimal trade is:

$$d = \frac{ts^*}{2Q\Gamma_d} - \frac{x_d+u}{2}. \tag{1}$$

In a separating equilibrium $s^{i*} = s^i = x_d^i/A_d$ for each $i=1,\ldots,m$. So, the presence of the dual trader is seen to have two opposite effects on an informed trader's incentive to trade. Suppose $x_d > 0$ (a buy order). If x_d is increased, the broker infers that the informed traders' information is improved and so s^* is higher as well. The broker trades more, d is higher and so is the resulting price. Thus, this signalling effect tends to inhibit informed traders from trading aggressively.

On the other hand, a higher x_d also reduces d from the second term in (1). This is a "second-mover disadvantage" for the broker as she has to accommodate market orders of any size by the informed and tends to encourage informed trades. For finite m, however, the signalling effect always dominates the second-mover effect, so that $B_1 > 0$ in equilibrium (x_d and d always have the same sign). The broker optimally mimics the trading decisions of the informed. Also, the broker optimally takes the opposite position of the aggregate uninformed trades.

Given (1), each informed trader i chooses x_d^i to maximize her conditional expected profits $E(I_d^1|s^i)$, where $I_d^i = \left(v - \Gamma_d d - \Gamma_d x_d^i - \Gamma_d \sum_{j \neq i} x_d^j - \Gamma_d u\right) x_d^i$. After incorporating the optimal value of d from equation (1) into I_d^i, the first-order condition for x_d^i, $i \neq j$ is:

$$\frac{t(1+Q)s^i}{2} = \Gamma_d\left[x_d^i + 0.5(m-1)E(x_d^i|s^i)\right] + \frac{ts^{i*}}{Q}. \tag{2}$$

Equation (2) says that the marginal value of an additional trade for the i-th informed trader is equal to its marginal cost. This cost has two components: the change in the price due to her own and her rivals' expected trades plus the change in the broker's inference as to her information. After using the facts that (i) $s^{i*} = s^i = x_d^i/A_d$ in equilibrium and (ii) $E(s^j|s^i) = ts^i$ for $j \neq i$, A_d is obtained as the coefficient of s^i in (2):

$$A_d = \frac{t^2(m-1)}{\Gamma_d Q(Q+1)}. \tag{3}$$

From (3), $A_d = 0$ when $m = 1$. But $A_d = 0$ cannot be a separating equilibrium since the functions $x_d^i = A_d s^i$, $i=1,\ldots,m$ are then no longer invertible.

Lemma 1: When m = 1, there is no solution to the dual trading model.

The result can be interpreted as follows. The inhibiting effect of the broker's piggybacking or mimicking behavior on any individual informed trader is inversely related to m, the number of informed customers the broker has. For m = 1, this inhibiting effect exactly offsets the marginal value of an extra trade for the individual informed trader as the first-order condition (2) reduces to:

$$t(s^i - s^{i\bullet}) = \Gamma_d x_d^i. \tag{4}$$

So, for any $x_d^i > 0$, the marginal cost of an additional trade for the informed always exceeds its marginal benefits.[9] Substituting (3) into (1), the optimal dual trading function is given by:

$$d = \frac{x_d}{t(m-1)} - \frac{u}{2}. \tag{5}$$

Finally, by using (3) and (5) in conjunction with the marketmaker's zero profit assumption the optimal value of market depth is derived as:

$$\Gamma_d = 2\,\frac{\sqrt{mt\Sigma_v}}{(1+Q)\sqrt{\Sigma_u}}. \tag{6}$$

Proposition 1 fully characterizes the dual trading equilibrium.

Proposition 1: If $m > 1$ and $t > 0$, there exists an unique solution to the dual trading model in which $x_d^i = A_d s^i$, $i=1,\ldots,m$, $d = B_1 x_d - \frac{u}{2}$ and $p_d = \Gamma_d y_d$ where A_d is given by (3), B_1 by the coefficient of x_d in (5) and Γ_d by (6).

What determines the extent of dual trading in the market? Define $ab(d) = ab(B_1 x_d - u/2)$ as the absolute value of dual trading volume. Fix m, the number of informed traders. The effect of increasing the information precision t is to make informed trades more sensitive to the information signals and so make the broker's observations more informative. This tends to increase ab(d). But, a higher t also increases informed trading volume x_d and this tends to reduce ab(d) via the second-mover effect. Thus, ab(d) is increasing in t only if $t(m-1) < 1$--i.e., if the total amount of information in the market is sufficiently low. From (5), $B_1 > 1$ when $t(m-1) < 1$ which means that a one share trade by an informed trader leads to a more than one share trade by the broker in the same direction.

Corollary 1: Sign $[\delta ab(d)/\delta t] = \text{sign}[B_1 - 1]$.

Next, fit t and consider the effect of increasing the number of informed traders m on dual trading volume ab(d). Suppose that the initial realizations are $(s^1,\ldots,s^m)$ and the (m+1)th informed trader observes a realization s^{m+1} from the same distribution. Now, the broker's observation of the trade of any individual informed trader is less valuable. But, at the same time, she observes more informed trades. The net effect on ab(d) depends on the sign of $s(m) = \sum_{i=1}^{m} s^i$ and the sign of s^{m+1}. If s(m) and s^{m+1} are of opposite signs, then ab(d) always decreases. If they are of the same sign, then ab(d) will increase if the additional information s^{m+1} is sufficiently valuable (in a sense made precise in the appendix).

Corollary 2: Suppose there are m informed traders with signal realizations $s^1,\ldots,s^m$ and with given information precision t. If an additional informed trader arrives with information realization s^{m+1} drawn from the original signal distribution, then the change in dual trading volume $\Delta d < 0$ if s^{m+1} and $s(m) = \sum_{i=1}^{m} s^i$ are of opposite signs. $\Delta d > 0$ if s(m) and s^{m+1} are of the same sign and s^{m+1} is large in magnitude.

II. THE MARKET IMPACT OF DUAL TRADING

In weighing the costs and benefits of dual trading, a regulator might be interested in its effect on aggregate market characteristics (total trading volume and profits, market depth and price efficiency) as well as its distributional effect on individual groups of market participants. These groups include the informed and uninformed traders and the broker. The distributional impact of dual trading may be discerned by considering its effects on the trading volumes of the informed and the uninformed,[10] the broker's commission fees and traders' expected profits net of commission costs. The impact of dual trading on aggregate market characteristics is studied in this section and the distributional question is analyzed in the following two sections.

A. The Nondual Trading Model

I will compare the dual trading solution obtained in Section I with the solution obtained when dual trading is completely banned. The broker is then a pure intermediary, bringing her customers' orders to the market.[11] The resulting trading game is a Cournot-Nash game in trading quantities. Each informed trader places an order x_n^i with the broker based on her information s^i. The broker submits the total order flow $y_n = x_n + u$ (where x_n is total informed trades in the nondual trading market) to the marketmaker for execution. The price determined is $p_n = \Gamma_n y_n$. Lemma 2 describes the nondual trading equilibrium.

Lemma 2: If there is no dual trading, a solution always exists provided $t > 0$. The informed traders trade $x_n^i = A_n s^i$ and the price is $p_n = \Gamma_n y_n$, where:

$$A_n = \frac{t}{\Gamma_n(1+Q)}, \quad \Gamma_n = \frac{\sqrt{mt\Sigma_v}}{(1+Q)\sqrt{\Sigma_u}}. \tag{7}$$

B. <u>Trading Volume and Gross Profits, Market Depth and Price Efficiency</u>

Due to "piggybacking" by the broker, it is reasonable to expect that $x_d < x_n$. The difference in informed trading volume depends upon the trading intensities A_n and A_d, as well as the market depths Γ_d and Γ_n. By inspection of (6) and (7), $\Gamma_d = 2\Gamma_n$: relative to the market without dual trading, market depth is half its value with dual trading. This reflects the fact that the dual trading broker offsets half of the uninformed trading volume. Given this result, the difference in informed trading volume:

$$x_n - x_d = \frac{ts}{\Gamma_d Q} \qquad (8)$$

which is positive for t > 0. ts/Q represents what the broker learns about the unknown v from observing the m-vector of informed trades. The more informative in this observation, the greater is the relative shrinkage of informed trading volume in the dual trading market. The difference in informed trading volume is also positively related to market depth, since a deeper market allows the broker to trade larger amounts with less concern about the price impact.

However, the broker herself provides an additional source of trading activity in the dual trading market. Considering the dual trading and informed trading activities together:

$$x_d + d = \frac{1}{\Gamma_d}\frac{ts}{1+Q} - \frac{u}{2} \tag{9}$$

$$x_n = \frac{1}{\Gamma_n}\frac{ts}{1+Q}. \tag{10}$$

From (9) and (10), any difference in the total trading volumes between the two markets occur because the broker offsets part of the noise trading. Otherwise, the market depth would be equal in the two markets and so would the total trading volumes. The broker's offsetting of noise trades has two effects in the dual trading market: first, total informed trading volume (by the informed traders <u>and</u> the broker) is halved due to the halving of market depth and, second, net noise trading (of the broker plus the noise traders) is halved. Hence, $y_n = 2y_d$: total trading volume is twice as high without dual trading.

Define price efficiency PI_i, $i = d,n$ as $\Sigma_v - var(v|p_i)$. It can be shown that $PI_i = (\Gamma_i)^2\Sigma_{y_i}$, where Σ_{y_i} is the variance of total trading volume in market $i = d,n$. PI_i depends positively on the volatility of total trading volume (since this depends positively on the informed trading intensities A_i) and inversely on market depth (since prices are less sensitive to volume, and hence to information, in deep markets). It follows that $PI_d = PI_n$ because, relative to the nondual trading market, the reduction in the variance of total trading volume is exactly offset by the reduction in market depth in the dual trading regime.

Let I_i denote the combined unconditional expected profits of the informed group (before observing any signals or paying any commissions) in the i-th market, $i = d,n$. Since the price level $p_i = \Gamma_i y_i$, it is the same in the two markets. It follows that $I_d < I_n$: gross profits of the informed are strictly lower with dual trading (since the price level is the same but informed trading volume is lower). Total trading profits in the dual trading market is $I_d + \pi$, where π is the broker's trading profits. Total <u>informed</u> trading profits in the dual trading market (I_d plus that part of the broker's profits obtained from mimicking informed trades) is half of I_n (since the price level is the same but total informed trading volume is half of x_n). But, the broker also profits from her observance of noise trading and the amount of this profit equals $\Gamma_d \Sigma_u/4$ or $\Gamma_n \Sigma_u/2$, which is exactly half of I_n. Thus $I_d + \pi = I_n$.

In equilibrium, the uninformed traders as a group suffer losses and the amount of their losses mirrors the total trading profits of the informed traders and the broker. Denote $L_g = I_d - I_n + \pi$ as the difference in the gross losses of the uninformed traders between the dual and nondual trading regimes. Therefore, $L_g = 0$.

Proposition 2: (1) $y_n = 2y_d$ and $\Gamma_d = 2\Gamma_n$. Total trading volume and market depth in the dual trading market are half their values in the nondual trading market. (2) $PI_d = PI_n$ and $I_d + \pi = I_n$. Gross trading profits and price efficiency are the same with or without dual trading. (3) $I_d < I_n$ and $L_g = 0$. Gross profits of the informed are lower with dual trading. Gross losses of the uninformed are unchanged.

In the following section, the basic model is extended to allow for rational behavior by the uninformed traders.

III. HEDGING BY UNINFORMED TRADERS

There are h risk-averse uninformed traders ("hedgers") who trade for purely risk-sharing reasons. The development of the model here follows Spiegel and Subrahmanyam (1992). Each hedger j has random endowment w^j, which is assumed to be normally distributed with mean zero and variance Σ_w. w^j, j=1,…,h are independent of each other and all other random variables in the model. All hedgers have negative exponential utility functions with risk-aversion parameter R.

Suppose that in market i all hedgers j=1,…,h submit market orders u_i^j to the broker and follow linear trading rules of the form $u_i^j = D_i w^j$. Let the total uninformed trading volume in market i be $u_i = \sum_{j=1}^{h} u_i^j$. If π_i^j is the profit of the j-th hedger in the i-th market, then u_i^j is chosen to maximize her utility or certainty-equivalent profits $G_i^j = E(\pi_i^j | w^j) - \frac{R}{2} Var(\pi_i^j | w^j)$. Let G_i, i = d,n be the sum of the utilities of all h hedgers in the i-th market. The informed traders and the broker's maximization problem remains the same as before, since each w^j is independent of v.[12] Market depth is now positively related to the magnitude of the "hedge factor" D_i[13] (since this increases the variance of the total order flow) and to the risk aversion parameter R. Further, the equilibrium $D_i < 0$ since the marginal utility of the hedgers from a purchase (sale) is negative if endowments are positive (negative). Lemma 3 describes the equilibrium for the nondual trading market. The appendix describes the dual trading equilibrium.

Lemma 3: An equilibrium to the hedger model exists if R satisfies equation (A15) in the appendix. In equilibrium, each hedger j=1,…,h trades $u_n^j = D_n w^j$, where $D_n < 0$, market depth is $1/\Gamma_n$, and:

$$-D_n\Gamma_n = \frac{\sqrt{mt\Sigma_v}}{(1+Q)\sqrt{h\Sigma_w}} \tag{11}$$

D_n is defined in (A14) of the appendix.

As in Spiegel and Subrahmanyam (1992), equilibrium exists if the amount of risk-aversion and noise in the market exceeds the amount of information available. This is the condition specified in (A15). From (11) and (A19) in the appendix, $\Gamma_d ab(D_d) = 2\Gamma_n ab(D_n)$, where $ab(D_i)$ denotes the absolute value of D_i. How does dual trading affect the choice of D_i? The price impact of liquidity trades is given by $\Gamma_i u_i$. Conjecture that $\Gamma_d = 2\Gamma_n$. Then, since net liquidity trades are halved with dual trading, the price impact of liquidity trades (including those by the broker) is identical in both markets and so $D_d = D_n$. This in turn means that $u_d = u_n$ and the initial conjecture regarding the market depths must be correct as well.[14] As before, the price levels are identical in the two markets and, so by implication, $G_d = G_n$. Hence, the results about the market impact of dual trading reached in the previous section is robust to the introduction of rational uninformed traders.

Proposition 3: $D_d = D_n$, $G_d = G_n$ and $\Gamma_d = 2\Gamma_n$. The hedgers' trading volumes and gross utilities are the same with or without dual trading. The depth of the market is halved when dual trading is permitted.

IV. A MODEL OF TRADING WITH COMMISSION FEES

Suppose that, in market i, the broker charges a per trade commission fee of $\$c_i$ to cover her costs of brokerage. Agents make their decisions in the following sequence. At stage zero, the broker determines the commission fee c_i. At stage one, the informed traders observe their signal realizations and c_i. Uninformed traders observe their endowment realizations and c_i. Then, both groups of traders decide how much to trade. At stage two, if dual trading is allowed, the broker's trade size is also contingent on c_d and her observance of informed and uninformed trades (if any). Finally, a price is set in the manner specified earlier. Stage zero is analyzed in Section V. The subsequent stages are analyzed here.

I will assume that the commission fees are fixed (independent of the order size) and have to be paid whenever a trader buys or sells a share of the asset. The optimal trading decisions are now based on traders' profits, net of her commission fees. Further, the equilibrium distributions of trades are likely to be truncated-normal because, for realizations of s^i and w^i close to zero, net profits may be negative if the commission fee is high. Thus, there could be a no-trade interval around zero (the mean value of the information signals and the endowments).[15] Anticipating that this conjecture will hold true, I will now introduce some useful results on the expectations and variances of a random variable defined with respect to the truncated normal distribution.[16]

Result 1: Let r be a random variable normally distributed with mean 0 and variance Σ_r. Let $[a_1,a_2]$ denote a closed interval. Define $n_1 = a_1/\sqrt{\Sigma_r}$ and $n_2 = a_2/\sqrt{\Sigma_r}$. Then:

1. $E(r|r < a_1 \text{ and } r > a_2) = \sqrt{\Sigma_r}\, M^r$ where $M^r = \dfrac{\phi(n_2) - \phi(n_1)}{1 + \Phi(n_1) - \Phi(n_2)}$, ϕ is the normal density and Φ is the normal distribution.
2. $\mathrm{Var}(r|r < a_1 \text{ and } r > a_2) = \Sigma_r\left[-(M^r)^2 + 1 + \dfrac{n_2\phi(n_2) - n_1\phi(n_1)}{1 + \Phi(n_1) - \Phi(n_2)}\right]$.

In particular, notice that if $a_2 = +a$ and $a_1 = -a$, then $n_2 = +n$ and $n_1 = -n$ and the two moments take the following special form:

1. $E(r|r < -a \text{ and } r > +a) = 0$.
2. $\mathrm{Var}(r|r < -a \text{ and } r > +a) = V^r = \Sigma_r\left[1 + \dfrac{n\phi(-n)}{\Phi(-n)}\right] > \Sigma_r$. Further, V^r is increasing in n.

I will first solve for the nondual trading model. Let N_n^i be the profits of i-th informed trader (net of c_n). Conjecture that the equilibrium trading strategy of the i-th informed trader is:

$$\begin{aligned} x_n^i &= A_n s^j + A_2 M^x + A_3 M^u \quad \text{if } E(N_n^i|s^i) > 0. \\ &= 0 \text{ otherwise.} \end{aligned} \tag{12}$$

(12) states that the equilibrium informed trades of the i-th informed trader depends upon three variables: her own information s^i, the expected trades of rival informed traders M^x and the expected uninformed trades M^u. Notice that, in the model without brokerage fees, these latter two variables were zero and so did not enter into the equilibrium trading strategies. However, since some trades may no longer be profitable because of the commission fees, M^x and M^u can potentially be non-zero.

For the uninformed traders suppose G_n^j is the utility of the j-th uninformed trader and that $\overline{G}$ is her reservation utility (i.e., her utility when she does not trade and does not pay the commission fee). Conjecture that the equilibrium trading strategy of the uninformed trader is:

$$\begin{aligned} u_n^j &= D_n w^j + D_2 M^x + D_3 M^u \quad \text{if } G_n^j > \overline{G} \\ &= 0 \text{ otherwise.} \end{aligned} \tag{13}$$

The i-th informed trader chooses x_n^i to maximize $E(N_n^i | s^i)$, where:

$$N_n^i = \left[v - \Gamma_n x_n^i - \Gamma_n \sum_{j \neq i} x_n^j - \Gamma_n \sum_{j=1}^{h} u_n^j \right] x_n^i - c_n. \tag{14}$$

Given the conjectures about the equilibrium trading volumes, it is shown in the appendix that $E(N_n^i | s^j) > 0$ requires that $\Gamma_n (x_n^i)^2 > c_n$. Therefore, the no-trade interval for the i-th informed trader (computed with respect to the trading volume x_n^i) is given by $N_n^x = \left[-\sqrt{\frac{c_n}{\Gamma_n}}, +\sqrt{\frac{c_n}{\Gamma_n}} \right]$. From Result 1, $M^x = 0$. Similarly, it is proved in the appendix that $M^u = 0$.

Since $M^x = 0$ and $M^u = 0$, whenever the profits of the informed traders are positive and the utilities of the uninformed traders exceed the reservation level, the form of the equilibrium trading strategies are the same as in the model without commission fees. The marketmaker sets a price whenever she observes a positive net order flow. What's different is the market depth, which now depends on the variance of the net order flow computed with respect to the truncated distributions of

x_n and u_n. Let V_n^v, V_n^s and V_n^v denote the variances of the random variables v, s^i and w^j (with respect to their truncated distributions) in the nondual market, as defined in Result 1.[17,18] The following proposition characterizes the equilibrium.

Proposition 4: Suppose dual trading is banned and all traders pay a fixed commission fee c_n. Suppose also that equations (A33) and (A34) in the appendix are satisfied. Then, in equilibrium each informed trader i trades $x_n^i = A_n s^i$ if $E(N_n^i | s^i) > 0$ and 0 otherwise, where A_n is defined in (7). Each uninformed trader trades $u_n^j = D_n w^j$ if $G_n^j > \bar{G}$ and 0 otherwise, where:

$$D_n \Gamma_n = -\sqrt{\frac{mtV_n^v}{hV_n^w}} \frac{1}{1+Q} \tag{15}$$

and D_n is defined in (A32) in the appendix.

Equations (A33) and (A34) together ensure that $D_n < 0$ in equilibrium. The reader can easily check that when $\Sigma_v = V_n^v$ and $\Sigma_w = V_n^w$ the equilibrium of Proposition 4 specializes to that in Lemma 3 (the case with no commission fees). The no-trade intervals for the informed (T_n^I) and uninformed traders (T_n^u) are:

$$T_n^I = \left[-a_n^I, +a_n^I\right] \text{ where } a_n^I = \frac{1+Q}{t} \sqrt{c_n \Gamma_n} \tag{16}$$

$$T_n^u = \left[-a_n^u, +a_n^u\right] \text{ where } a_n^u = \frac{1}{ab(D_n)} \sqrt{\frac{2c_n}{2\Gamma_n + RZ_n}} \tag{17}$$

where $Z_n > 0$ in equilibrium and is defined in (A29) of the appendix.

Both T_n^I and T_n^U increase in the commission fee. For the informed and uninformed traders, the direction of change is generally ambiguous. Corollary 3 states without proof (available from the author) the likely change in the informed traders' no-trade intervals with respect to the number of traders m and h and the information precision t. A noteworthy result is that the market depth may increase with the introduction of commission fees if the volatility of the uninformed traders' endowments increases more than the asset volatility.

Corollary 3: (1) T_n^U is increasing in c_n. If V_n^w is small it may also increase in m and t. If V_n^w is large, T_n^U may increase in h also. (2) T_n^I is increasing in c_n. If V_n^w is small, then T_n^I may increase in m and decrease in h. If hV_n^w is large and m is small, then it may decrease in t. (3) Suppose $\frac{V_n^v}{V_b^w} < \frac{\Sigma_v}{\Sigma_w}$. If the values of R, Σ_w and h are high and that of mt is low, then market depth and ab(D_n) may be higher with commission fees than without.

The analysis for the dual trading case proceeds along similar lines. Informed and uninformed traders do not trade for some values of their information and endowments. Equilibrium exists if either at least two informed traders trade (see Proposition 1) or an uninformed trader trades. The broker either offsets half of the uninformed trading volume or piggybacks on informed trading volume or both. An interesting aspect of this equilibrium (as well as the nondual trading solution) is that a solution exists even when only uninformed or only informed traders actually trade. So long as trading occurs, the forms of the trading strategies and the pricing rule remain the same. The trading volume and price levels are different because the volatility of the equilibrium trading volumes are different. The following lemma summarizes the results for the dual trading case.

Lemma 4: Suppose dual trading is permitted and the broker charges a fee of c_d per trade. An equilibrium exists if (A41) in the appendix is satisfied. Further, when $x_d > 0$, at least two informed traders must trade. In equilibrium, the i-th informed trader trades $x_d^i = A_d s^i$ if $E(N_d^i | s^j) > 0$ where A_d is defined in (3), and 0 otherwise. The j-th uninformed trader trades $u_d^j = D_d w^j$ if her utility from trading $G_d^j > \overline{G}$, her reservation utility, and is 0 otherwise. D_d is defined in (A40) of the appendix. The broker trades $d = B_1 x_d - \frac{u_d}{2}$ whenever $x_d > 0$ and/or $u_d > 0$ and 0 otherwise. $B_1 = 1/(Q-1)$. The price level $p_d = \Gamma_d y_d$ whenever $y_d > 0$, where:

$$\Gamma_d D_d = -2 \frac{\sqrt{mtV_d^v}}{\sqrt{hV_d^w}} \frac{1}{1+Q}. \tag{18}$$

Denote T_d^I and T_d^u as the no-trading intervals of the informed and uninformed traders, respectively. Then:

$$T_d^I = \left[-a_d^I, +a_d^I\right] \text{ where } a_d^I = \frac{1+Q}{t} \frac{\sqrt{Q}}{\sqrt{Q-1}} \sqrt{c_d \Gamma_d} \tag{19}$$

$$T_d^u = \left[-a_d^u, +a_d^u\right] \text{ where } a_d^u = \frac{1}{ab(D_d)} \sqrt{\frac{c_d}{\Gamma_d + RZ_d}} \tag{20}$$

where Z_d is defined in (A39) of the appendix. Thus dual trading affects the probability of a customer trade, whether informed or informed.

V. THE EFFECT OF DUAL TRADING ON COMMISSION FEES

Suppose that the broker faces a fixed cost k_0 and a variable cost k_1 of conducting business, both costs being non-negative. To be consistent with the representation of the commission fee, it is assumed that k_1 is incurred on a per trade basis. Thus, the broker's total variable costs in market i equals k_1 times the expected number of informed and uninformed trades in market i. Following Fishman and Longstaff (1992), I assume that the brokerage business is competitive and so the commission fee c_i is chosen so that the broker's expected trading profits π plus her expected commission income equals zero. Then the broker's commission fees with and without dual trading must satisfy:

$$k_0 - \pi = 2(c_d - k_1)\left[m\Phi(-a_d^I) + h\Phi(-a_d^u)\right] \tag{21}$$

$$k_0 = 2(c_n - k_1)\left[m\Phi(-a_n^I) + h\Phi(-a_n^u)\right]. \tag{22}$$

In (21), the broker's fixed cost k_0 equals her trading profits plus the net profit margin per trade (c_d-k_1) times the expected number of customer trades.[19] For each group of customers, informed and uninformed, the expected number of trades equals the number of traders (m and h) times the probability of a trade. The probability of a trade for a particular trader group is calculated as twice the value of the normal distribution at the lower end point of the no-trade interval for that group. Equation (22) has a similar interpretation, except that the broker has no trading profits by hypothesis.

Equations (21) and (22) define c_d and c_n implicitly. An explicit solution for the equilibrium commission fees is not possible.[20] Instead, I will establish sufficient conditions such that $(c_d - c_n)$ can be signed. If the expected number of customer trades is the same or greater with dual trading then clearly $c_d < c_n$, as in Fishman and Longstaff (1992). However, if the expected number of customer trades is lower with dual trading, k_0 is large (the precise condition is in the appendix) and dual trading profits are small in magnitude, then $c_d > c_n$ is possible. The dual trading profits are:

$$\pi = \frac{-D_d}{2Q}\sqrt{mthV_d^v V_d^w}. \tag{23}$$

Simulation results show that the RHS of (23) is decreasing in m and increasing in h and V_d^w. Thus dual trading profits will tend to be small when there are many informed traders and relatively few uninformed traders who do not trade aggressively. The broker may then be able to offset the loss of her trading profits when dual trading is banned through higher commission income generated by the greater number of customer trades and thus maintain lower commission fees c_n. By combining (21) and (22), the following relation obtains:

$$\begin{aligned} &2(m+h)\left[c_n\left(\Phi(-a_n^I)+\Phi(-a_n^u)\right)-c_d\left(\Phi(-a_d^I)+\Phi(-a_d^u)\right)\right] \\ &\quad = \pi + 2k_1\left[m\left(\Phi(-a_n^I)-\Phi(-a_d^I)\right)+h\left(\Phi(-a_n^u)-\Phi(-a_d^u)\right)\right]. \end{aligned} \tag{24}$$

(24) is in the nature of an accounting relation which says that the difference in expected commission costs of all customers between the two markets (the LHS of the equation) is equal to the dual trading profits plus the difference in the expected variable costs incurred by the broker. Thus, if the probability of a customer trade is lower with dual trading then expected commission costs of all customers are also lower. Note that this holds even if $c_d > c_n$.

Proposition 5: (1) $c_d < c_n$ if the probability of all trades is the same or higher with dual trading. (2) If the probability of all trades is lower with dual trading, the broker's fixed costs k_0 are large and the magnitude of dual trading profits is small, $c_d > c_n$ is possible. However, when the probability of all trades is lower with dual trading the expected commission costs of all traders is also lower with dual trading.

An alternative characterization of the sign of $(c_d - c_n)$ in terms of just the probability of informed trading and the change in the market depth parameter is given below.

Corollary 4: Assume that there exist solutions c_d and c_n to (21) and (22). $c_d < c_n$ if the probability of an informed trade is higher with dual trading and the market depth is never higher, i.e., $\Gamma_d \geq \Gamma_n$. $c_d > c_n$ if the probability of an informed trade is lower and market depth is sufficiently higher with dual trading, specifically $\Omega\Gamma_d \leq (\Omega-1)\Gamma_n$.

The expected net trading profits of all informed traders in market i is denoted by N_i. These are equal to I_i, the expected gross profits (conditional on trading) minus the expected commission costs:

$$N_n = \frac{mtV_n^v}{\Gamma_n(1+Q)^2} - 2mc_n\Phi\left(-a_n^I\right) \tag{25}$$

$$N_d = \frac{mtV_d^v}{\Gamma_d(1+Q)^2}\,\frac{Q-1}{Q} - 2mc_d\Phi\left(-a_n^u\right). \tag{26}$$

Expected gross profits I_i (the first terms on the RHS of the two equations) are calculated with respect to the truncated distributions of x_i. Expected commission costs are equal to the commission fees c_i times m, the number of informed traders times the probability of an informed trade in the i-th market. Without commission fees, $I_d < I_n$. Here:

$$I_d - I_n = \frac{\sqrt{mth}}{1+Q}\left[\frac{-D_d(Q-1)}{2Q}\sqrt{V_d^vV_d^w} + D_n\sqrt{V_n^vV_n^w}\right]. \tag{27}$$

Thus $I_d > I_n$ is possible if $ab(D_d)\sqrt{V_d^vV_d^w} > 2\sqrt{V_n^vV_n^w}\,ab(D_n)$, i.e., the product of the asset volatility and uninformed trading volatility is twice as high with dual trading and, further, the amount of information mt is large (making the ratio $(Q-1)/Q$ closer to 1). For $I_d < I_n$, it is sufficient that the product of the volatilities is lower with dual trading. Combining (24), (25) and (26) gives:

$$N_n - N_d = I_n - (I_d+\pi) - 2h\left[c_d\Phi(-a_d^u) - c_n\Phi(-a_n^u)\right] \\ -2k_1\left[m\left(\Phi(-a_n^I) - \Phi(-a_d^I)\right) + h\left(\Phi(-a_n^u) - \Phi(-a_d^u)\right)\right] \tag{28}$$

where

$$I_d + \pi - I_n = \frac{\sqrt{mth}}{1+Q}\left[-D_d\sqrt{V_d^v V_d^w} + D_n\sqrt{V_n^v V_n^w}\right]. \tag{29}$$

The difference in net profits of the informed depends upon the difference between gross profits of all informed traders (including the broker), the difference in commission costs paid by the uninformed and, finally, the change in the broker's variable costs. For the uninformed traders, the appendix shows that:

$$E(G_n) - E(G_d) = I_n - (I_d+\pi) + Rh\left[Z_n\frac{(D_n)^2}{2} - Z_d(D_d)^2\right] \\ + 2h\left[c_d\Phi(-a_d^u) - c_n\Phi(-a_n^u)\right]. \tag{30}$$

In words, the difference in the expected utilities of the uninformed between the two markets depends upon the difference in expected gross profits of the informed (including that of the dual trader), the difference in the expected volatility of uninformed trading profits and the relative commission costs. The following proposition summarizes the results on the gross and net profits (utilities) of informed (uninformed) traders.

Proposition 6. Suppose $V_d^v < V_n^v$, $V_d^w < V_n^w$ and $(V_d^v/V_d^w) > (V_n^v/V_n^w)$. Then (1) $I_d < I_n$ and $(I_d+\pi) < I_n$. Further, the utilities (gross of commission fees) of the uninformed traders are also lower with dual trading.

(2) $N_d < N_n$ if, in addition, the expected commission costs of the

uninformed traders are lower and the broker's total variable costs are higher with dual trading. (3) $E(G_d) < E(G_n)$ with the further condition that the expected commission costs of the uninformed traders are higher with dual trading.

Except for the inequality relating I_d and I_n, all other inequalities are reversed if the signs on the sufficient conditions are reversed as well. Proposition 6 suggests that for the informed traders to be worse off with dual trading, two things must happen. First, uninformed traders pay relatively less in commission costs with dual trading. Second, the probability of an informed trade must increase less than the probability of an uninformed trade with dual trading. This is because, from Result 1, for a random variable r the volatility v^r is increasing in the size of the no-trade interval.

For the uninformed traders, those conditions which are sufficient to ensure that $I_d + \pi$ exceed (fall short of) I_n are also sufficient to ensure that the expected volatility of uninformed trading profits is higher (lower) with dual trading. thus, dual trading changes the gross utilities of the uninformed traders in the same direction as the gross profits of all the other traders in the market. It is easy to show, however, that the net utilities are more likely to increase with dual trading if the relative commission costs of the informed traders increase.

V. CONCLUSION

The paper considers the effects of allowing brokers to trade on their own account (to dual trade) in addition to their usual intermediary function. In the model without commission fees, dual trading leads to a reduction in total trading volumes and market depth, but price efficiency is not affected. The dual trading activity comes at the expense of both informed and uninformed traders who are the broker's customers. Informed traders are hurt because they are forced to be less aggressive in anticipation of the broker mimicking their trades and "piggybacking" on their information. The broker also takes the opposite position of uninformed trades and offsets a portion of uninformed trading. However, the price impact of an uninformed trade and the variance of uninformed profits are unchanged and so, on balance, uninformed traders neither gain nor lose with dual trading.

The model is extended to include fixed (i.e., independent of the order size) commission fees which traders must pay to the broker if they choose to trade. Now, traders sometimes choose not to trade if their expected profits are less than the commission fee. The effect of dual trading depends upon the changes in the probabilities of informed and uninformed trades. Commission fees may be higher with dual trading if the probability of a customer trade decreases and the size of dual trading profits is small. However, the expected commission costs of all traders are also lower with dual trading when the probability of a customer trade is lower.

Ignoring commission fees, informed gross profits are lower and expected utilities of the uninformed are the same with dual trading. With commission fees, when the uninformed traders pay relatively lower commission costs with dual trading and the probability of their trading increases relative to that of the informed traders then informed traders net profits are lower. For the uninformed traders their utilities gross of commission fees increase when the gross profits of all other traders increase as well. Their net utilities will increase with dual trading if, in addition, their relative commission costs are lower with dual trading.

FOOTNOTES

[1]See The Chicago Tribune, February 4, 1992 and The Wall Street Journal, February 7, 1992 for details.

[2]Later, the basic model is extended to allow for rational behavior by uninformed traders.

[3]The broker is assumed to have no private information of her own. For a model with a privately informed broker, see Sarkar (1991).

[4]The assumption of a batch market maximizes the negative impact of piggybacking on informed trading. But the effect would remain in a setting where the orders of the customers and the broker are executed (and priced) separately, so long as some subset of the informed customers make repeat purchases or sales via the same broker. From Kyle (1985), the optimal dynamic trading strategy of an informed trader is in fact to dribble her trades over time.

[5]By virtue of being able to infer the information of her informed customers, the broker can be considered to be a de facto informed trader.

[6]I will adopt the convention of labelling the decision variables of individual agents with a superscript and market variables with a subscript. The subscript d will refer to the solution in the dual trading model and the subscript n to the nondual trading solution.

[7]The broker is assumed to have no independent information regarding v. In a previous version of this paper (Sarkar (1991)), the broker has her own information but does not observe uninformed trades u.

Also m = 1. This makes for some interesting interactions between the information of the single insider and that of the broker. For example, for low precision of the broker's information, the insider's trades is actually decreasing in the precision of her own information!

[8]For m = 2, I have checked that the results are unchanged if the informed traders have information of different precisions. I conjecture that this is true for general m.

[9]In Sarkar (1991), an equilibrium exists even with m = 1 so long as the precision of the broker's information is positive. The reason is that, if the broker has an independent source of information about v, she attaches relatively less weight to her observation of the informed trade. The change in the broker's inference, when the informed trader buys or sells an extra share, no longer fully offsets the marginal value of that extra share traded.

[10]Since, in the model so far, the trading volume of the uninformed is not a choice variable, the impact of dual trading on uninformed trading volume will not be considered until Section III (when I do model the uninformed trading decision).

[11]This interpretation of a nondual trading market as one where the broker does not trade at all appears to be consistent with market realities (for example, the S&P 500 index futures market where such a ban is currently effective). In the previous version of this paper (Sarkar (1991)) the broker was assumed to be independently informed and this raised troubling issues as to what happens to the broker's information when dual trading is banned.

A related issue concerns the choice of brokers. Some brokers can commit not to dual trade (as occurs in reality). Those customers who value the superior trading skills of dual trading brokers (as suggested in Grossman (1989)) will continue to patronize them. Others, perhaps more concerned with frontrunning and other potential abuses, may choose the brokers committed not to dual trade. Thus, my model should be seen as a reduced form of this more general situation where traders and brokers are matched according to their varying needs. I thank the Editor, Douglas W. Diamond, for bringing these points to my attention.

[12] Of course, the actual informed and dual trading volumes will be different since market depth will be different, in general.

[13] The "hedge factor" is different from the hedge ratio familiar in the futures/options literature since, here, the asset being used for hedging purposes is the same one that is being hedged.

[14] Note that this is precisely the situation considered in the previous section, where u was exogenously fixed at the same level in the two markets. The appendix proves this result formally.

[15] I wish to thank the anonymous referee for pointing this out to me.

[16] The best source for these results is Johnson and Kotz (1970).

[17] Given the equilibrium relationships between informed trades and the information signals, and uninformed trades and the endowments, the truncated distributions of x_n and u_n imply (in equilibrium) truncated distributions for v, s^j and w^i as well. It is sometimes more intuitive to express the results in terms of these exogenous variables rather than

the endogenous trading variables. Notice also that since the no-trade intervals are specific to each market i, so are these variances.

[18]Define $t_n = (V_n^v/V_n^s)$ as the information precision defined with respect to the truncated distribution. Since the no-trade interval is the same for each informed trader, the information precision defined with respect to the untruncated distribution $t = t_n$.

[19]The broker also derives commission income from her own trades but, since this is also a cost to the broker for the same amount, it balances out in the broker's account.

[20]The reason is that the normal distributions must be evaluated at the lower end point of the no-trade intervals of the traders. These no-trade intervals depend upon the market depth which, in turn, depend upon the variances of v and w defined with respect to their truncated distributions. From Result 1, however, the variances themselves are functions of the normal distributions. This circularity prevents an explicit expression of the equilibrium commission fees.

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APPENDIX

Proof of Lemma 1

Let $E(v|s^{i*},\ldots,s^{m*}) = as^*$, where $s^* = \sum_{i=1}^{m} s^{i*}$. Applying Bayes' rule, $a = \dfrac{1/\Sigma_e}{1/\Sigma_v + m/\Sigma_e} = \dfrac{t}{1+(m-1)t}$, where $t = \Sigma_v/(\Sigma_v+\Sigma_e)$. This gives the optimal dual trade $d(s^*,x_d)$ as given in (1).

Incorporating (1), each informed trader i's profits I_d^i can be written as:

$$I_d^i = \left(v - \frac{t}{1+(m-1)t}\frac{s^*}{2} - \frac{\Gamma_d}{2}x_d - \frac{\Gamma_d u}{2}\right)x_d^i \tag{A1}$$

Substituting $s^{i*} = s^i = \dfrac{x_d^i}{A_d}$ for each $i = 1,\ldots,m$ into (A1):

$$\begin{aligned} E\left(I_d^i|s^i\right) = \Bigg[ts^i &- \frac{x_d^i}{2}\left(\Gamma_d + \frac{t}{1+(m-1)t}\frac{1}{A_d}\right) \\ &- \sum_{j\neq i}\frac{E\left(x_d^j|s^i\right)}{2}\left(\Gamma_d + \frac{t}{1+(m-1)t}\frac{1}{A_d}\right)\Bigg]x_d^i \end{aligned} \tag{A2}$$

Substituting $E(x_d^j|s^i) = A_d t s^i$ for each $j \neq i$ into (A2) and then differentiating with respect to x_d^i gives (2). When m = 1, (2) has the form:

$$x_d^i\left(\frac{t}{A_d} + \Gamma_d\right) = ts^i \tag{A3}$$

It is easily checked that there is no $A_d > 0$ such that $x_d^i = A_d s^i$ has a solution.

Proof of Proposition 1

From (3) and (5), $y_d = x_d + d + u = \dfrac{x_d[1+t(m-1)]}{t(m-1)} + \dfrac{u}{2}$, or:

$$y_d = \frac{ts}{[2+t(m-1)]} \cdot \frac{1}{\Gamma_d} + \frac{u}{2}, \text{ where } s = \sum_{i=1}^{m} s^i \tag{A4}$$

(6) follows from solving Γ_d = covariance (v,y_d)/variance (y_d).

Proof of Corollary 1

From (3), (5) and (6), $ab(d) = ab\left(\dfrac{\sqrt{t\Sigma_u}}{2Q\sqrt{m\Sigma_v}}s - \dfrac{u}{2}\right)$. Ignoring terms independent of m and t, $\dfrac{\delta ab(d)}{\delta t} = \dfrac{1-t(m-1)}{4Q^2\sqrt{mt}}ab(s) > 0$ for $t(m-1) < 1$ or $B_1 > 1$, from (1).

Proof of Corollary 2

Define $Q(m) = 1 + t(m-1)$, $Q(m+1) = 1 + tm$, $s(m) = \sum_{i=1}^{m} s^i$ and $s(m+1) = \sum_{i=1}^{m+1} s^i$. Notice that $Q(m+1) > Q(m)$. Then:

$$\Delta d = ab(d(m+1)) - ab(d(m))$$

$$= \frac{\sqrt{t\Sigma_u}}{2\sqrt{\Sigma_v}}\left[\frac{1}{\sqrt{1+m}} \cdot \frac{1}{Q(m+1)} \cdot ab(s(m+1)) - \frac{1}{\sqrt{m}} \cdot \frac{1}{Q(m)} \cdot ab(s(m))\right].$$

If $s(m) > 0$, then

$$\Delta d = \frac{\sqrt{t\Sigma_u}}{2\sqrt{\Sigma_v}} \cdot \left[\frac{1}{\sqrt{1+m}} \cdot \frac{1}{Q(m+1)} - \frac{1}{\sqrt{m}} \cdot \frac{1}{Q(m)}\right]s(m)$$

$$+ \frac{s^{m+1}}{2} \cdot \frac{\sqrt{\Sigma_u}}{\sqrt{\Sigma_v}} \cdot \frac{1}{\sqrt{m+1}} \cdot \frac{1}{Q(m+1)} < 0 \text{ if } s^{m+1} < 0$$

$$> 0 \text{ if } s^{m+1} > ks(m)$$

where $k = \frac{\sqrt{1+m}}{\sqrt{m}} \cdot \frac{Q(m+1)}{Q(m)}$.

If s(m) < 0, then

$$\Delta d = \frac{\sqrt{t\Sigma_u}}{2\sqrt{\Sigma_v}}\left[\frac{1}{\sqrt{1+m}} \cdot \frac{1}{Q(1+m)} - \frac{1}{\sqrt{m}} \cdot \frac{1}{Q(m)}\right]ab(s(m))$$

$$- \frac{s^{m+1}}{2} \cdot \frac{\sqrt{t\Sigma_u}}{\sqrt{\Sigma_v}} \cdot \frac{1}{\sqrt{1+m}} \cdot \frac{1}{Q(1+m)} < 0 \text{ if } s^{m+1} > 0.$$

If $s^{m+1} < 0$, then $\Delta d > 0$ if $ab(s^{m+1}) > kab(s(m))$, where k is defined earlier.

Proof of Lemma 2

Each informed trader "i" chooses x_n^i to maximize $E(I_n^i|s^i)$, where:

$$I_n^i = \left(v - \Gamma_n x_n^i - \Gamma_n \sum_{j\neq i} x_n^j\right) x_n^i \tag{A5}$$

The first-order condition for x_n^i yields:

$$x_n^i = \frac{ts^i}{2\Gamma_n}\left[1-(m-1)\Gamma_n A_n\right] \tag{A6}$$

Solving (A6) yields the equilibrium value for A_n. Solving for Γ_n in the usual way, (7) is obtained.

Proof of Proposition 2

Price informativeness $PI_i, i = d,n$ is defined as:

$$PI_i = \Sigma_v - \text{Variance}(v|p_i) = \Gamma_i^2 \Sigma_{y_i}. \tag{A7}$$

Since $\Gamma_d = 2\Gamma_n$ but $y_d = \frac{y_n}{2}$, $PI_d = PI_n$.

$I_i = E([v-p_i]x_i)$ is the total unconditional expected profits of the informed traders in market i = d,n. $I_d < I_n$ since $p_d = p_n$ and $x_d < x_n$.

$$I_d + \pi = E([v-p_d]\,[x_d+d])$$
$$= E\left\{[v-p_n]\left[\frac{x_n}{2} - \frac{u}{2}\right]\right\} = \frac{I_n}{2} + \frac{\Gamma_n\Sigma_u}{2}. \tag{A8}$$

The result follows because $\Gamma_n\Sigma_u = \frac{\sqrt{mt\Sigma_v\Sigma_u}}{1+Q} = I_n$.

<u>Proofs of Lemma 3 and Proposition 3</u>

Consider the nondual market first. The profits of the j-th hedger:

$$\pi_n^j = v\left(u_n^j + w^j\right) - \Gamma_n u_n^j\left(u_n^j + D_n\sum_{m\neq j} w^m + x_n\right). \tag{A9}$$

From the maximization problem of the informed traders, $\Gamma_n x_n = \frac{ts}{1+Q}$. Substituting for $\Gamma_n x_n$ in (A9) and taking expectations:

$$E\left(\pi_n^j|w^j\right) = -\Gamma_n\left(u_n^j\right)^2 \tag{A10}$$

$$\mathrm{Var}\left(\pi_n^j|w^j\right) = E\left[vw^j + u_n^j\left(v - \frac{ts}{1+Q} - \Gamma_n D_n\sum_{m\neq j} w^m\right)\right]^2$$
$$= \Sigma_v(w^j)^2 + \left(u_n^j\right)^2\left[\Sigma_v\left(1 - \frac{mt(Q+2)}{(Q+1)^2}\right) + (\Gamma_n D_n)^2(h-1)\Sigma_w\right] \tag{A11}$$
$$+ 2\Sigma_v u_n^j w^j\,\frac{(2-t)}{1+Q}.$$

(A10) and (A11) together define G_n^j. Differentiating G_n^j with respect to u_n^j:

$$-2\Gamma_n u_n^j - Ru_n^j\left[\Sigma_v\left(1 - \frac{mt(Q+2)}{(Q+1)^2}\right) + (\Gamma_n D_n)^2(h-1)\Sigma_w\right] - R\Sigma_v w^j\frac{(2-t)}{1+Q} = 0. \tag{A12}$$

Equating D_n with the coefficient of w^j in (A12) gives:

$$R(D_n)^3(\Gamma_n)^2(h-1)\Sigma_w + D_n\left[2\Gamma_n + R\Sigma_v\left(1 - \frac{mt(Q+2)}{(Q+1)^2}\right)\right] + R\Sigma_v\frac{(2-t)}{Q+1} = 0. \tag{A13}$$

Solving for Γ_n, I get equation (11) in the text. It follows from equation (11) that since $\Gamma_n > 0$ to satisfy the second-order condition for the informed traders, $D_n < 0$ in equilibrium. Substituting for $\Gamma_n D_n$ in (A13) and solving for D_n:

$$D_n = \frac{2\sqrt{mt\Sigma_v/(h\Sigma_w)} - R\Sigma_v(2-t)}{R\Sigma_v(2-t) - \dfrac{R\Sigma_v mt}{h(Q+1)}} \tag{A14}$$

Since $D_n < 0$, equilibrium exists if:

$$R\Sigma_v(2-t) > \frac{2\sqrt{mt\Sigma_v}}{\sqrt{h\Sigma_w}} \tag{A15}$$

The denominator of D_n in (A14) is always positive, so (A15) is sufficient for $D_n < 0$. To show this, rewrite the denominator as:

$$\text{Den} = R\Sigma_v\left[1-\frac{mt(Q+2)}{(Q+1)^2}\right] + \frac{(h-1)mt}{h(Q+1)^2}$$

$$= \frac{R\Sigma_v}{(Q+1)^2}[(1-t)(4+mt)+t^2] + \frac{(h-1)mt}{h(Q+1)^2}$$

$$> 0.$$

In the second step, the definition $Q = 1 + t(m-1)$ has been used.

Now, consider the dual trading market.

$$\pi_d^j = v(u_d^j + w^j) - \frac{\Gamma_d u_d^j}{2}\left(u_d^j + D_d\sum_{m\neq j} w^m\right) - \frac{ts}{1+Q}u_d^j. \qquad \text{(A16)}$$

Proceeding in the usual way,

$$E(\pi_d^j|w^j) = -\frac{\Gamma_d}{2}(u_d^j)^2 \qquad \text{(A17)}$$

$$\text{Var}(\pi_d^j|w^j) = \Sigma_v(w^j)^2 + (u_d^j)^2\left[\Sigma_v\left(1-\frac{mt(2+Q)}{(1+Q)^2}\right) + \frac{(\Gamma_d D_d)^2}{4}(h-1)\Sigma_w\right] + 2\Sigma_v w^j u_d^j \frac{(2-t)}{1+Q} \qquad \text{(A18)}$$

$$\Gamma_d D_d = -2\frac{\sqrt{mt}\sqrt{\Sigma_v}}{(1+Q)\sqrt{h\Sigma_w}}. \qquad \text{(A19)}$$

From (11) and (A19), $\Gamma_d D_d = 2\Gamma_n D_n$. Solving For D_d proves $D_d = D_n$. From these results, $E(\pi_d^j|w^j) = E(\pi_n^j|w^j)$ and $\text{Var}(\pi_d^j|w^j) = \text{Var}(\pi_n^j|w^j)$ and so $G_d^j = G_n^j$ for each j. Thus $G_d = G_n$. Finally, $D_d = D_n$ implies $\Gamma_d = 2\Gamma_n$ from (11) and (A19).

Proof of Proposition 4

$$E(N_n^j|s^j) = \left[ts^j - \Gamma_n x_n^j - \Gamma_n \sum_{i \neq j} E(x_n^i|s^j) - \Gamma_n \sum_{i=1}^{h} E(u_n^i|s^j) \right] x_n^j - c_n \tag{A20}$$

where

$$E(x_n^i|s^j) = tA_n s^j + (1-t)M_n^x \tag{A21}$$

$$E(u_n^i|s^j) = M_n^u \tag{A22}$$

$t = \Sigma_v/\Sigma_s$ and M_n^x, M_n^u are the truncated means defined in Result 1 (putting $r = x_n^i, u_n^i$. Note that, since $var(s^j) = \Sigma_s$, $j=1,\ldots,m$ and $var(w^i) = \Sigma_w$, $i=1,\ldots,h$, M_n^x and M_n^u are the same for each informed and uninformed trader in the nondual trading market).

Incorporating (A21) and (A22) into (A20) and differentiating with respect to x_n^j gives the following first-order condition:

$$2\Gamma_n x_n^j = ts^j - (m-1)\Gamma_n[tA_n s^j + (1-t)M_n^x] - h\Gamma_n M_n^u. \tag{A23}$$

Using (A23), the condition $E(N_n^j|s^j) > 0$ simplifies to:

$$\Gamma_n (x_n^j)^2 > c_n. \tag{A24}$$

So, from Result 1, $M^x = 0$. For the j-th uninformed trader:

$$\pi_n^j = v(u_n^j + w^j) - \Gamma_n u_n^j \left(u_n^j + D_n \sum_{m \neq j} w^m + x_n \right) - c_n$$

$$E(\pi_n^j|w^j) = -\Gamma_n(u_n^j)^2 - (h-1)\Gamma_n M^u u_n^j - m\Gamma_n M^x u_n^j - c_n$$

$$\mathrm{Var}(\pi_n^j|w^j) = \Sigma_v(w^j)^2 + (u_n^j)^2\left[\Sigma_v + (h-1)(\Gamma_n D_n)^2 V_n^w + \frac{mt^2}{(1+Q)^2}V_n^s + \frac{m(m-1)t^2}{(1+Q)^2}V_n^v - \frac{2mt}{1+Q}V_n^v\right] + 2u_n^j w^j \Sigma_v$$

where V_n^v, V_n^s and V_n^w are the truncated variances defined in Result 1, putting $r = v$, s^j and w^j. The optimal uninformed trade solves the following equation:

$$-2\Gamma_n u_n^j - (h-1)\Gamma_n M^u - m\Gamma_n M^x - Ru_n^j Z_n - Rw^j\Sigma_v = 0 \tag{A25}$$

where

$$Z_n = \Sigma_v + (h-1)(\Gamma_n D_n)^2 V_n^w + \frac{mt^2}{(1+Q)^2}V_n^s + \frac{m(m-1)t^2}{(1+Q)^2}V_n^v - \frac{2mt}{1+Q}V_n^v. \tag{A26}$$

The reservation utility $\bar{G} = -\frac{R}{2}\Sigma_v(w^j)^2$. Using (A25), the gains from trade $GT_n = G_n^j - \bar{G}$ are:

$$\begin{aligned} GT_n &= \frac{u_n^j}{2}[-2\Gamma_n u_n^j - 2(h-1)\Gamma_n M^u - 2m\Gamma_n M^x - RZ_n u_n^j - 2Rw^j\Sigma_v] - c_n \\ &= (u_n^j)^2\left[\Gamma_n + \frac{R}{2}Z_n\right] - c_n. \end{aligned} \tag{A27}$$

Thus, the condition $GT_n > 0$ requires:

$$u_n^j > \pm\frac{\sqrt{c_n}}{\sqrt{\Gamma_n + \frac{R}{2}Z_n}}. \tag{A28}$$

Again, from Result 1, $M^u = 0$.

A sufficient condition for the denominator in (A28) to be positive is that $Z_n > 0$. When $V_n^v \leq \Sigma_v$, this is always the case. Since $V_n^v > \Sigma_v$, however, it is possible that $Z_n < 0$. Rewrite Z_n as follows (using the fact that $t = t_n = V_n^v/V_n^s$:

$$Z_n = \Sigma_v - \frac{mtV_n^v}{(1+Q)^2} \cdot [\frac{1}{h} + 1 + Q]. \tag{A29}$$

Then $Z_n > 0$ provided:

$$\Sigma_v(1+Q) > mtV_n^v + \frac{mtV_n^v}{h(1+Q)}. \tag{A30}$$

Comparing with (A33) below, this is precisely one of the conditions needed for $D_n < 0$ in equilibrium. So, whenever equilibrium exists, $Z_n > 0$ must hold.

Putting $M^x = 0$ and $M^u = 0$ in (A23) and solving, I get $x_n^j = A_n s^j$ when $E(N_n^j | s^j) > 0$, where A_n is defined in (7). Next, solve for Γ_n:

$$\Gamma_n = \frac{mA_nV_n^v}{m(A_n)^2V_n^s + m(m-1)(A_n)^2V_n^v + h(D_n)^2V_n^w}$$

or

$$D_n\Gamma_n = -\frac{\sqrt{mtV_n^v}}{(1+Q)\sqrt{hV_n^w}}. \tag{A31}$$

Substituting $M^x = 0$ and $M^u = 0$ in (A25) and solving for D_n, I get a cubic equation in D_n. After substituting for $\Gamma_n D_n$ from (A31):

$$D_n = \frac{\dfrac{\sqrt{mtV_n^v}}{\sqrt{hV_n^w}} + RV_n^v mt - R\Sigma_v(1+Q)}{R\Sigma_v(1+Q) - RV_n^v mt - \dfrac{RV_n^v mt}{(1+Q)h}}. \tag{A32}$$

For equilibrium to exist, we need $\Gamma_n > 0$ and $D_n < 0$. To maintain consistency with the no commission fee case, I require the denominator of D_n to be positive and the numerator to be negative. The twin conditions are:

$$Z_n > 0 \tag{A33}$$

and

$$R\sqrt{mtV_n^v V_n^w} > \sqrt{h}\,(1+Q). \tag{A34}$$

To derive the no-trade intervals T_n^I and T_n^u, note that (A24) can be rewritten in the following way by using the definition of A_n from (7):

$$(ts^j)^2 > (1+Q)^2 c_n \Gamma_n. \tag{A35}$$

Thus, trade occurs if $ab(s^j) < a_n^I$, where a_n^I is defined in (16).

For the uninformed traders, since $u_n^j = D_n w_n^j$, trade occurs if (using (A27)):

$$(D_n w^j)^2 > \frac{2c_n}{2\Gamma_n + RZ_n} \tag{A36}$$

or $ab(w^j) > a_n^u$, where a_n^u is defined in (17).

<u>Proof of Lemma 4</u>

In equilibrium, $E(N_d^j|s^j) = (x_d^j)^2 \dfrac{\Gamma_d Q}{Q-1} - c_d$. Thus, the j-th informed trader trades provided:

$$ab(s^j) > \frac{1}{A_d} \cdot \frac{\sqrt{c_d}}{\sqrt{\Gamma_d}} \cdot \sqrt{\frac{Q-1}{Q}} = \frac{1+Q}{t} \cdot \sqrt{c_d \Gamma_d} \cdot \sqrt{\frac{Q}{Q-1}} \tag{A37}$$

using the definition of A_d from (3).

For the j-th uninformed trader, the gains from trade:

$$(G_d^j - \bar{G}) = (u_d^j)^2 [\Gamma_d + RZ_d] - c_d \tag{A38}$$

where

$$Z_d = \Sigma_v - \frac{mtV_d^v}{(1+Q)^2}\left[\frac{1}{h} + 1 + Q\right]. \tag{A39}$$

The j-th uninformed trader trades if:

$$ab(w^j) > \frac{1}{ab(D_d)} \cdot \frac{\sqrt{c_d}}{\sqrt{\Gamma_d + RZ_d}},$$

where

$$D_d = \frac{\dfrac{\sqrt{mtV_d^v}}{\sqrt{hV_d^w}} - R\Sigma_v(1+Q) + RV_d^v mt}{R\Sigma_v(1+Q) - RV_d^v mt - \dfrac{RV_d^v mt}{h(1+Q)}}. \tag{A40}$$

For $D_d < 0$, it is sufficient that $Z_d > 0$ and $R\sqrt{mtV_d^v V_d^w} > \sqrt{h}\,(1+Q)$. (A41)

Consider the broker's problem. If $x_d \geq 0$ and $u_d \geq 0$, then she trades $d = \frac{x_d}{t(m-1)} - \frac{u_d}{2}$. Further, from Proposition 1, there must be at least two informed trades when $x_d > 0$.

Proof of Proposition 5

(1) Let $\Phi(i) = m\Phi(-a_i^I) + h\Phi(-a_i^u)$, , i=d,n be the probability of all trades in market i. By hypothesis, $\Phi(d) \geq \Phi(n)$. Rewrite (21) and (22) as:

$$c_n = k_1 + \frac{k_0}{2\Phi(n)}$$

$$c_d = k_1 + \frac{k_0}{2\Phi(d)} - \pi.$$

Since $\pi > 0$, $c_d < c_n$.

(2) $$c_d - c_n = \frac{k_0}{2}\left(\frac{1}{\Phi(d)} - \frac{1}{\Phi(n)}\right) - \pi$$

$$> 0 \quad \text{if} \quad k_0 > \frac{2\pi\Phi(d)\Phi(n)}{\Phi(n) - \Phi(d)} \quad \text{and} \quad \Phi(n) > \Phi(d).$$

From (24), if $a_d^I > a_n^I$ and $a_d^u > a_n^u$, then the RHS of the equation is positive. Thus, the difference in expected commission costs of all traders between the nondual and dual trading market (the LHS of (24)) is positive.

Proof of Corollary 4

By hypothesis, $a_d^I < a_n^I$, or:

$$\frac{1+Q}{t}\frac{\sqrt{Q}}{\sqrt{Q-1}}\sqrt{c_d\Gamma_d} < \frac{1+Q}{t}\sqrt{c_n\Gamma_n}.$$

Since $\frac{Q}{(Q-1)} > 1$ and $\Gamma_d \geq \Gamma_n$, $c_d < c_n$ or there is a contradiction. If $a_d^I > a_n^I$, but $Q\Gamma_d \leq (Q-1)\Gamma_n$, we must have $c_d > c_n$ to avoid a contradiction.

<u>Proof of Proposition 6</u>

From the definition of Z_i, $ab(D_i)$ can be written as:

$$ab(D_i) = \frac{R\Sigma_v(1+Q) - \frac{\sqrt{mtV_i^v}}{\sqrt{hV_i^w}} - RV_i^v mt}{RZ_i(1+Q)}. \tag{A42}$$

Differentiate (A42) with respect to V_i^v and consider the numerator of this derivative:

$$RZ_i(1+Q)\left(\frac{Rmt}{h(1+Q)} - \frac{1}{2}\frac{\sqrt{mt}}{\sqrt{hV_i^v V_i^w}}\right) + Rmt\left(\frac{1}{h(1+Q)} + 1\right)\left(\frac{RV_n^v mt}{h(1+Q)} - \frac{\sqrt{mtV_i^v}}{\sqrt{hV_i^w}}\right)$$

> 0 because $Z_i > 0$ and (A34) and (A41) hold in equilibrium. Thus, $ab(D_i)$ is increasing in V_i^v. Clearly it is also decreasing in (V_i^v/V_i^w). So, $ab(D_d) < ab(D_n)$ if $V_d^v < V_n^v$ and $(V_d^v/V_d^w) > (V_n^v/V_n^w)$. If, in addition, $V_d^w < V_n^w$ then $I_d < I_n$. Further, $(I_d+\pi) < I_n$ from (29).

Using (A27) and (A38)

$$E(G_n) - E(G_d) = hE(u_n^j)^2\left[\Gamma_n + \frac{RZ_n}{2}\right] - 2c_n\Phi(-a_n^u)h$$
$$- hE(u_d^j)^2[\Gamma_d + RZ_d]\; 2c_d\Phi(-a_d^u)h \tag{A43}$$
$$= -\frac{D_n\sqrt{V_n^v V_n^w}\sqrt{mth}}{1+Q} + \frac{D_d\sqrt{V_d^v V_d^w}\sqrt{mth}}{1+Q} + \frac{hRZ_n(D_n)^2}{2}$$
$$- hRZ_d(D_d)^2 + 2h[\Phi(-a_d^u)c_d - \Phi(-a_n^u)c_n].$$

The second equality is derived from the first by observing that $u_i^j = D_i w^j$, i=d,n and by substituting in the value of $-(\Gamma_i D_i)$ from equations (15) and (18) in the text. From (29) in the text:

$$E(G_n) - E(G_d) = I_n - (I_d+\pi) + Rh\left[\frac{Z_n(D_n)^2}{2} - Z_d(D_d)^2\right] \tag{A44}$$
$$+ 2h[c_d\Phi(-a_d^u) - c_n\Phi(-a_n^u)]$$

which is equation (30) in the text.

From before, $(I_d+\pi) < I_n$ if $V_d^v < V_n^v$, $V_d^w < V_n^w$ and $(V_d^v/V_d^w) > (V_n^v/V_n^w)$. It remains to show that these conditions also ensure that:

$$\left[\frac{hRZ_n(D_n)^2}{2} - hRZ_d(D_d)^2\right] > 0.$$

I will refer to the numerator of $ab(D_i)$ in (A42) as (Num_i). Then,

$$hRZ_i(D_i)^2 = h(ab(D_i)) \cdot \frac{(Num_i)}{1+Q}$$

$$= \frac{h}{R} \frac{(Num_i)^2}{Z_i},$$

by repeated use of the definition of $ab(D_i)$ (from (A44)). The derivative of this last expression with respect to V_i^v is:

$$\frac{h}{R(Z_i)^2} \cdot \left[2Z_i(Num_i)\left(Rmt - \frac{1}{2} \cdot \frac{\sqrt{mt}}{\sqrt{hV_i^v V_i^w}}\right) + (Num_i)^2\left(\frac{mt}{1+Q} + \frac{mt}{h(1+Q)^2}\right)\right]$$

> 0 since $Z_i > 0$ and $Rmt > \frac{\sqrt{mt}}{\sqrt{hV_i^v V_i^w}}$ from (A34). Further, (Num_i) is decreasing in (V_i^v/V_i^w). Thus, under the given conditions, ignoring the commission fees $E(G_n) > E(G_d)$. This proves part (1) of the proposition.

Given our results from part (1), parts (2) and (3) follow directly from equations (28) and (30) in the text.